QCD
at
hadron colliders

Vittorio Del Duca
INFN Torino

Bhubaneswar 5 January 2006

QCD

- an unbroken Yang-Mills gauge field theory featuring asymptotic freedom and confinement
- in non-perturbative regime (low Q^2) many approaches: lattice, Regge theory, χ PT, large N_c, HQET
- in perturbative regime (high Q^2) QCD is a precision toolkit for exploring Higgs & BSM physics
- LEP was an electroweak machine
- Tevatron & LHC are QCD machines

Precision QCD

Precise determination of

- strong coupling constant α_s
- parton distributions
- electroweak parameters
- LHC parton luminosity

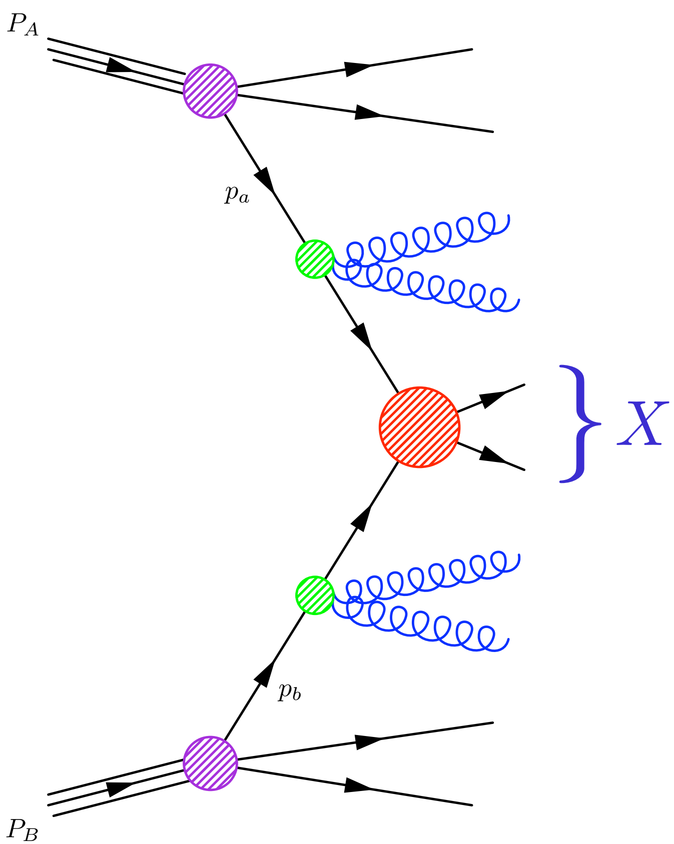
Precise prediction for

- Higgs production
- new physics processes
- their backgrounds

Strong interactions at high Q^2

- Parton model
- Perturbative QCD
  - factorisation
  - universality of IR behaviour
  - cancellation of IR singularities
  - IR safe observables: inclusive rates
    - jets
    - event shapes

Factorisation



is the separation between
the short- and the long-range interactions

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2 \, f_{a/A}(x_1, \mu_F^2) \, f_{b/B}(x_2, \mu_F^2)$$
$$\times \quad \hat{\sigma}_{ab \to X} \left(x_1, x_2, \{p_i^\mu\}; \alpha_S(\mu_R^2), \alpha(\mu_F^2), \frac{Q^2}{\mu_R^2}, \frac{Q^2}{\mu_F^2} \right)$$

$$X = W, Z, H, Q\bar{Q}, \text{high-}E_T\text{jets}, \ldots$$

$\hat{\sigma}$ is known as a fixed-order expansion in α_S

$$\hat{\sigma} = C\alpha_S^n (1 + c_1 \alpha_S + c_2 \alpha_S^2 + \ldots)$$

$c_1 =$ NLO $\qquad$ $c_2 =$ NNLO

or as an all-order resummation

$$\hat{\sigma} = C\alpha_S^n [1 + (c_{11} L + c_{10}) \alpha_S + (c_{22} L^2 + c_{21} L + c_{20}) \alpha_S^2 + \ldots]$$

where $L = \ln(M/q_T), \ln(1-x), \ln(1/x), \ln(1-T), \ldots$

$c_{11}, c_{22} =$ LL $\qquad$ $c_{10}, c_{21} =$ NLL $\qquad$ $c_{20} =$ NNLL

Factorisation-breaking contributions

- underlying event (see Rick Field's studies at CDF)
- power corrections
  - MC's and theory modelling of power corrections laid out and tested at LEP where they provide an accurate determination of α_S models still need be tested in hadron collisions (see e.g. Tevatron studies at different $\sqrt{s}$) ⇨
- diffractive events ⇨

Is double-parton scattering breaking factorisation ?

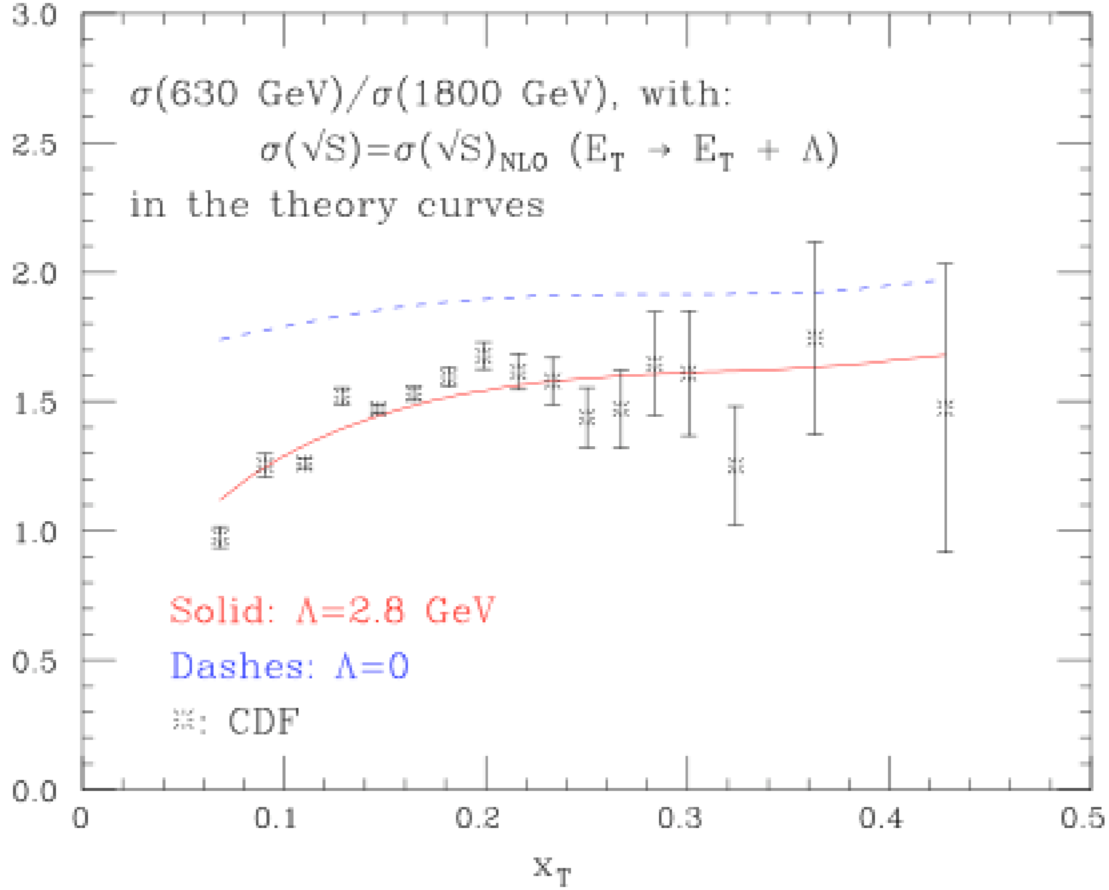
- observed by Tevatron CDF in the inclusive sample

  $p\bar{p} \to \gamma + 3$ jets

  potentially important at LHC $\sigma_D \propto \sigma_S^2$

Power corrections at Tevatron

Ratio of inclusive jet cross sections at 630 and 1800 GeV



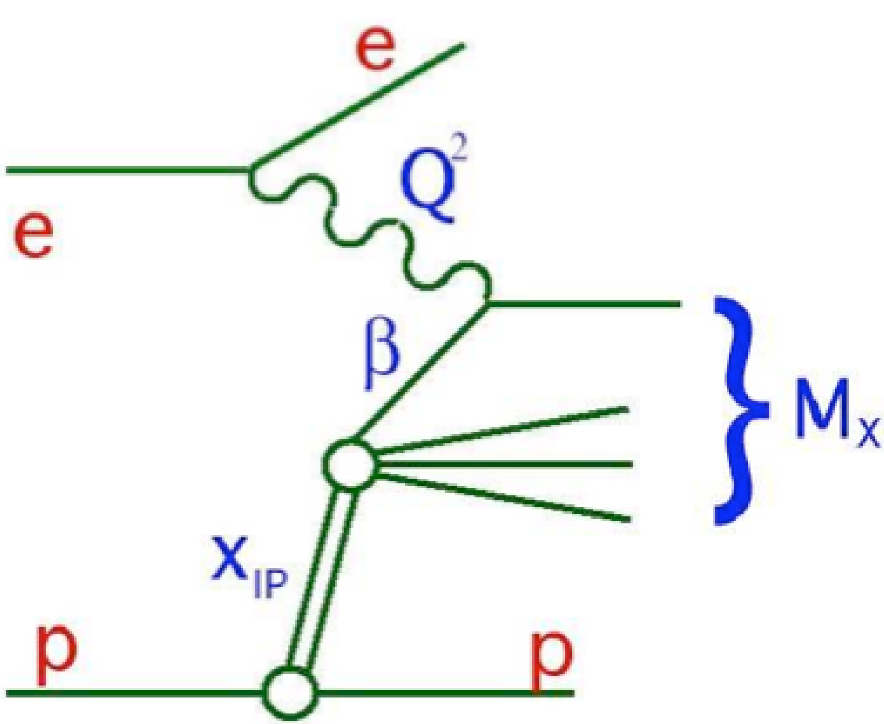
M.L. Mangano
KITP collider conf 2004

Bjorken-scaling variable

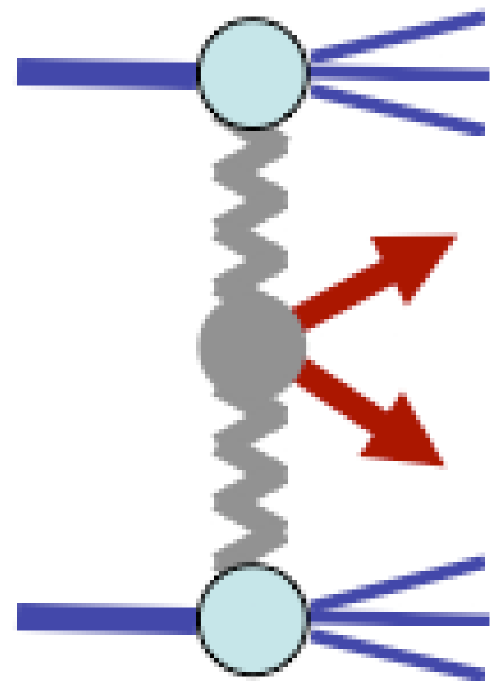
$$x_T = \frac{2E_T}{\sqrt{s}}$$

- In the ratio the dependence on the pdf's cancels
- dashes: theory prediction with no power corrections
- solid: best fit to data with free power-correction parameter Λ in the theory

Factorisation in diffraction ??

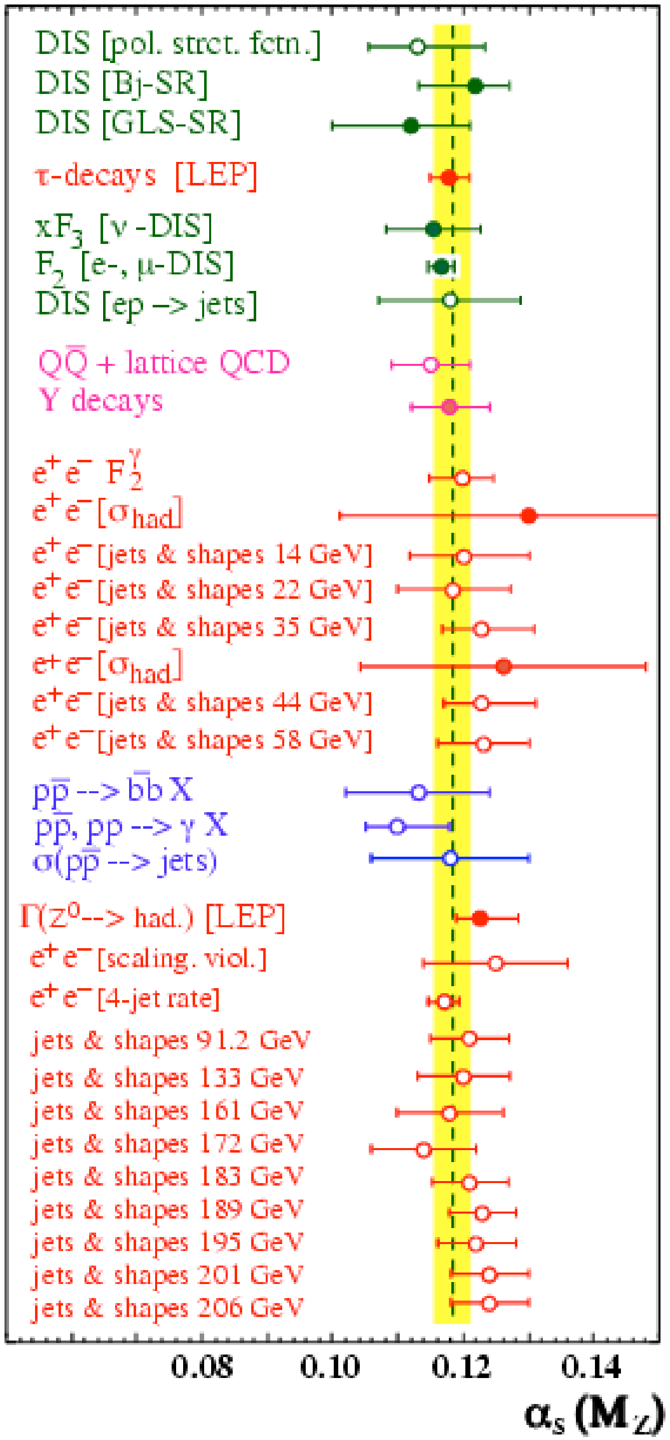


diffraction in DIS



double pomeron exchange in $p\bar{p}$

- no proof of factorisation in diffractive events
- data do not support it



filled symbols are NNLO results

Summary of $\alpha_S(M_Z)$

S. Bethke hep-ex/0407021

world average of $\alpha_S(M_Z)$
using $\overline{\mathrm{MS}}$ and NNLO results only

$$\alpha_S(M_Z) = 0.1182 \pm 0.0027$$

(cf. 2002 $\alpha_S(M_Z) = 0.1183 \pm 0.0027$
outcome almost identical
because new entries wrt 2002
- LEP jet shape observables and
4-jet rate, and HERA jet rates
and shape variables - are NLO)

3 complementary approaches to $\hat{\sigma}$

	matrix-elem MC's	fixed-order x-sect	shower MC's
final-state description	hard-parton jets. Describes geometry, correlations, ...	limited access to final-state structure	full information available at the hadron level
higher-order effects: loop corrections	hard to implement: must introduce negative probabilities	straightforward to implement (when available)	included as vertex corrections (Sudakov FF's)
higher-order effects: hard emissions	included, up to high orders (multijets)	straightforward to implement (when available)	approximate, incomplete phase space at large angles
resummation of large logs	?	feasible (when available)	unitarity implementation (i.e. correct shapes but not total rates)

M.L. Mangano KITP collider conf 2004

Matrix-element MonteCarlo generators

- multi-parton generation: processes with many jets (or W/Z/H bosons)
  - ALPGEN M.L.Mangano M. Moretti F. Piccinini R. Pittau A. Polosa 2002
  - MADGRAPH/MADEVENT W.F. Long F. Maltoni T. Stelzer 1994/2003
  - COMPHEP A. Pukhov et al. 1999
  - GRACE/GR@PPA T. Ishikawa et al. K. Sato et al. 1992/2001
  - HELAC C. Papadopoulos et al. 2000
- processes with 6 final-state fermions
  - PHASE E. Accomando A. Ballestrero E. Maina 2004
- merged with parton showers
  - all of the above, merged with HERWIG or PYTHIA
  - SHERPA F. Krauss et al. 2003

Shower MonteCarlo generators

- HERWIG B. Webber et al. 1992

  being re-written as a C++ code (HERWIG++)

- PYTHIA T. Sjostrand 1994

and more

- CKKW S. Catani F. Krauss R. Kuhn B. Webber 2001

  a procedure to interface parton subprocesses with
  a different number of final states to parton showers

- MC@NLO S. Frixione B. Webber 2002

  a procedure to interface NLO computations to shower MC's

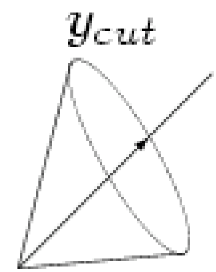
NLO features

- Jet structure: final-state collinear radiation
- PDF evolution: initial-state collinear radiation
- Opening of new channels
- Reduced sensitivity to fictitious input scales: μ_R, μ_F
  - predictive normalisation of observables
    - first step toward precision measurements
    - accurate estimate of signal and background for Higgs and new physics
- Matching with parton-shower MC's: MC@NLO

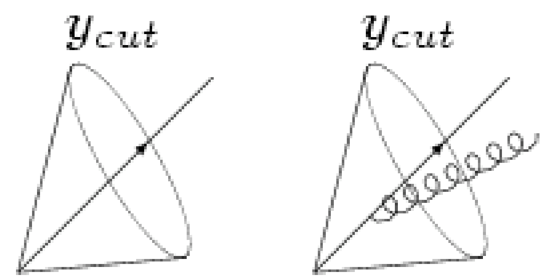
Jet structure

the jet non-trivial structure shows up first at NLO

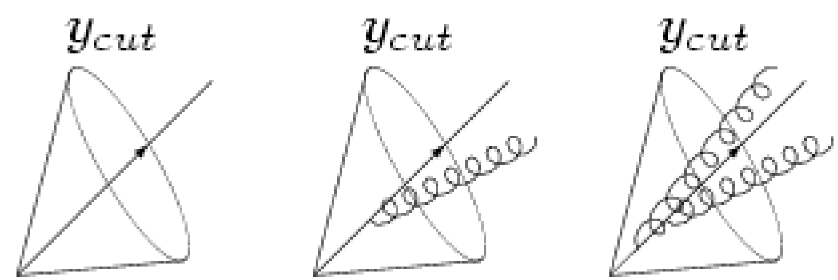
leading order



NLO



NNLO



Desired NLO cross sections

Run II Monte Carlo Workshop, April 2001

Single boson	Diboson	Triboson	Heavy flavour
$W + \leq 5j$	$WW + \leq 5j$	$WWW + \leq 3j$	$t\bar{t} + \leq 3j$
$W + b\bar{b} + \leq 3j$	$WW + b\bar{b} + \leq 3j$	$WWW + b\bar{b} + \leq 3j$	$t\bar{t} + \gamma + \leq 2j$
$W + c\bar{c} + \leq 3j$	$WW + c\bar{c} + \leq 3j$	$WWW + \gamma\gamma + \leq 3j$	$t\bar{t} + W + \leq 2j$
$Z + \leq 5j$	$ZZ + \leq 5j$	$Z\gamma\gamma + \leq 3j$	$t\bar{t} + Z + \leq 2j$
$Z + b\bar{b} + \leq 3j$	$ZZ + b\bar{b} + \leq 3j$	$WZZ + \leq 3j$	$t\bar{t} + H + \leq 2j$
$Z + c\bar{c} + \leq 3j$	$ZZ + c\bar{c} + \leq 3j$	$ZZZ + \leq 3j$	$t\bar{b} + \leq 2j$
$\gamma + \leq 5j$	$\gamma\gamma + \leq 5j$		$b\bar{b} + \leq 3j$
$\gamma + b\bar{b} + \leq 3j$	$\gamma\gamma + b\bar{b} + \leq 3j$		
$\gamma + c\bar{c} + \leq 3j$	$\gamma\gamma + c\bar{c} + \leq 3j$		
	$WZ + \leq 5j$		
	$WZ + b\bar{b} + \leq 3j$		
	$WZ + c\bar{c} + \leq 3j$		
	$W\gamma + \leq 3j$		
	$Z\gamma + \leq 3j$		

NLO history of final-state distributions

- $e^+e^- \to 3$ jets — K. Ellis, D. Ross, A. Terrano 1981
  $e^+e^- \to 4$ jets — Z. Bern et al., N. Glover et al., Z. Nagy Z. Trocsanyi 1996-97
- $pp \to 1, 2$ jets — K. Ellis J. Sexton 1986, W. Giele N. Glover D. Kosower 1993
  $pp \to 3$ jets — Z. Bern et al., Z. Kunszt et al. 1993-1995, Z. Nagy 2001
- $pp \to \gamma\gamma$ — B. Bailey et al 1992, T. Binoth et al 1999
  $pp \to \gamma\gamma + 1$ jet — Z. Bern et al. 1994, V. Del Duca et al. 2003
- $pp \to V + 1$ jet — W. Giele N. Glover & D. Kosower 1993
  $pp \to V + 2$ jet — Bern et al., Glover et al. 1996-97, K. Ellis & Campbell 2003
- $pp \to Vb\bar{b}$ — K. Ellis & J. Campbell 2003
- $pp \to VV$ — Ohnemus & Owens, Baur et al. 1991-96, Dixon et al. 2000
- $pp \to Q\bar{Q}$ — Dawson K. Ellis Nason 1989, Mangano Nason Ridolfi 1992
  $pp \to Q\bar{Q} + 1$ jet — A. Brandenburg et al. 2005-6 ?
- $pp \to H + 1$ jet — C. Schmidt 1997, D. De Florian M. Grazzini Z. Kunszt 1999
- $pp \to H + 2$ jets (WBF) — Campbell, K. Ellis; Figy, Oleari, Zeppenfeld 2003
- $pp \to HQ\bar{Q}$ — W. Beenakker et al. ; S. Dawson et al. 2001
- $e^+e^- \to 4$ fermions — Denner Dittmaier Roth Wieders 2005

NLOJET++

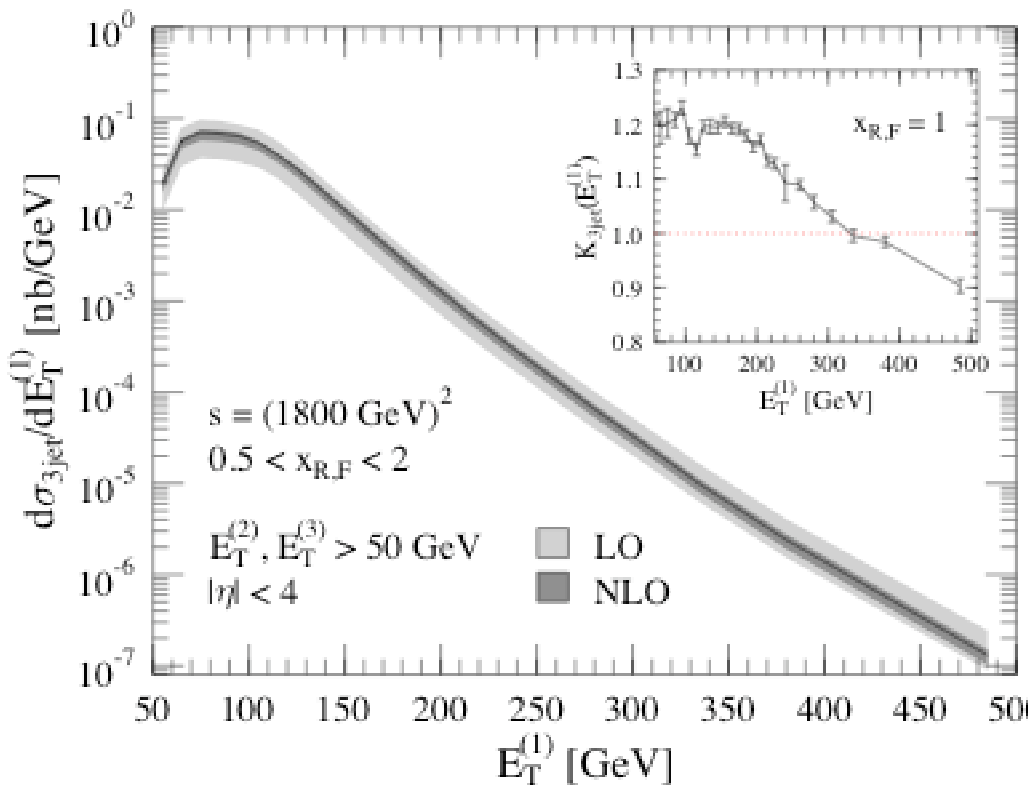
Author(s): Z. Nagy
http://www.ippp.dur.ac.uk/~nagyz/nlo++.html
Multi-purpose C++ library for calculating jet cross-sections in e^+e^- annihilation, DIS and hadron-hadron collisions.

$e^+e^- \longrightarrow \leq 4$ jets

$ep \longrightarrow (\leq 3+1)$ jets

$p\bar{p} \longrightarrow \leq 3$ jets

$k_\perp$ algorithm



hep-ph/0110315

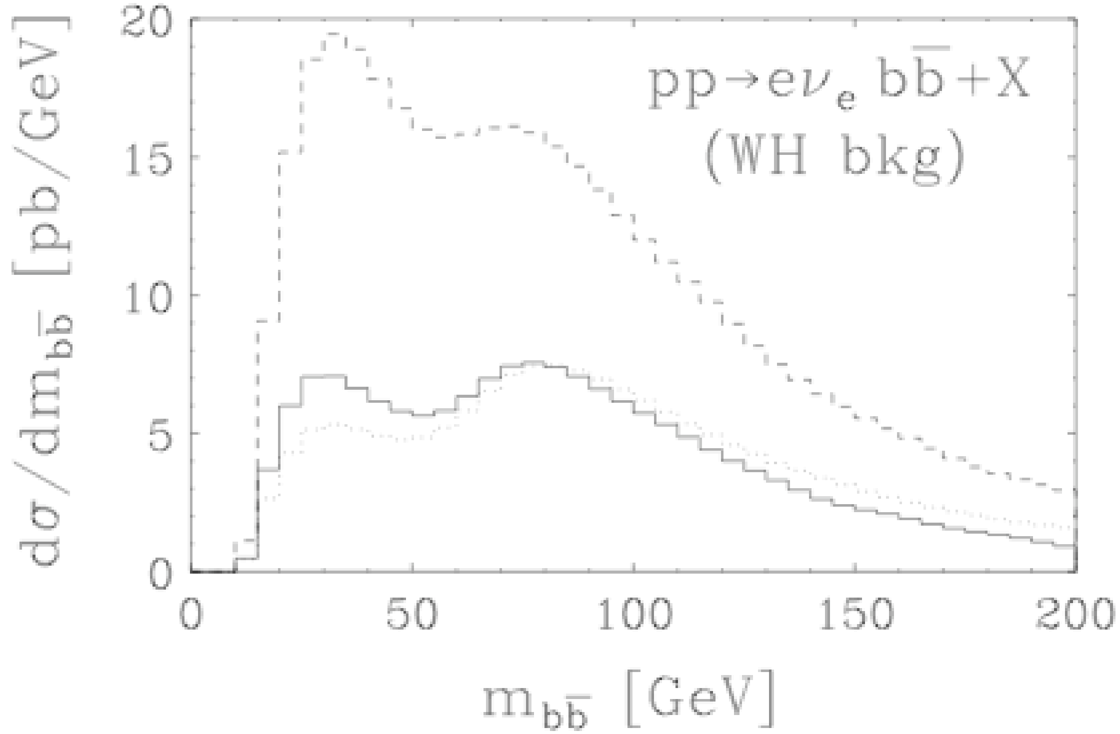
MCFM

Author(s): JC, R. K. Ellis
http://mcfm.fnal.gov
Fortran package for calculating a number of processes involving vector bosons, Higgs, jets and heavy quarks at hadron colliders.

$p\bar{p} \longrightarrow V+ \leq 2$ jets

$p\bar{p} \longrightarrow V + b\bar{b}$

with $V = W, Z$.

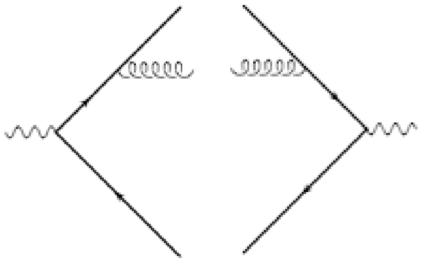


hep-ph/0308195

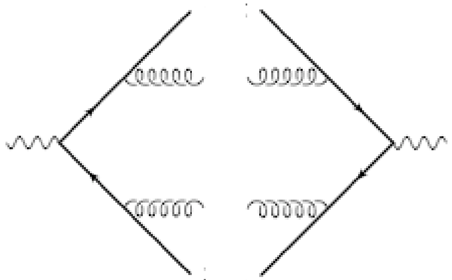
NLO assembly kit

$e^+e^- \to 3$ jets

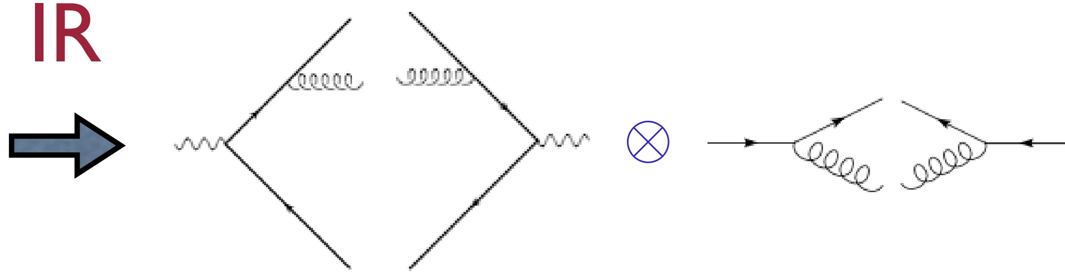
leading order $\qquad |\mathcal{M}_n^{tree}|^2$



NLO real



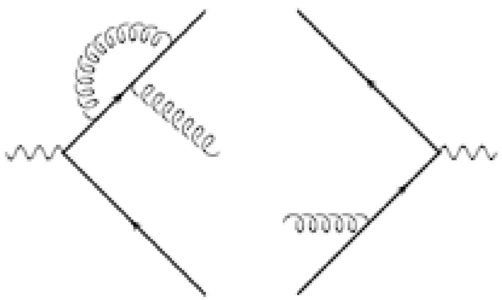
IR



$$|\mathcal{M}_{n+1}^{\text{tree}}|^2 \quad \to \quad |\mathcal{M}_n^{\text{tree}}|^2 \times \int dPS |P_{\text{split}}|^2$$

$$= -\left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon}\right)$$

NLO virtual



$d = 4 - 2\epsilon$

$$\int d^d l \; 2(\mathcal{M}_n^{loop})^* \mathcal{M}_n^{tree} \;=\; \left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon}\right) |\mathcal{M}_n^{tree}|^2 + fin.$$

NLO production rates

Process-independent procedure devised in the 90's

- slicing — Giele Glover & Kosower
- subtraction — Frixione Kunszt & Signer; Nagy & Trocsanyi
  - dipole — Catani & Seymour
  - antenna — Kosower; Campbell Cullen & Glover

$$\sigma = \sigma^{\rm LO} + \sigma^{\rm NLO} = \int_m d\sigma^B_m \; J_m + \sigma^{\rm NLO}$$

$$\sigma^{\rm NLO} = \int_{m+1} d\sigma^{\rm R}_{m+1} J_{m+1} + \int_m d\sigma^{\rm V}_m J_m$$

the 2 terms on the rhs are divergent in d=4

use universal IR structure to subtract divergences

$$\sigma^{\rm NLO} = \int_{m+1} \left[d\sigma^{\rm R}_{m+1} J_{m+1} - d\sigma^{\rm R,A}_{m+1} J_m \right] + \int_m \left[d\sigma^{\rm V}_m + \int_1 d\sigma^{\rm R,A}_{m+1} \right] J_m$$

the 2 terms on the rhs are finite in d=4

Observable (jet) functions

J_m vanishes when one parton becomes soft or collinear to another one

$$J_m(p_1, ..., p_m) \to 0\,, \quad \text{if} \quad p_i \cdot p_j \to 0$$

⇨ $d\sigma_m^{\mathrm{B}}$ is integrable over 1-parton IR phase space

J_{m+1} vanishes when two partons become simultaneously soft and/or collinear

$$J_{m+1}(p_1, ..., p_{m+1}) \to 0\,, \quad \text{if} \quad p_i \cdot p_j \text{ and } p_k \cdot p_l \to 0 \quad (i \neq k)$$

R and V are integrable over 2-parton IR phase space

observables are IR safe

$$J_{n+1}(p_1, .., p_j = \lambda q, .., p_{n+1}) \to J_n(p_1, ..., p_{n+1}) \quad \text{if} \quad \lambda \to 0$$
$$J_{n+1}(p_1, .., p_i, .., p_j, .., p_{n+1}) \to J_n(p_1, .., p, .., p_{n+1}) \quad \text{if} \quad p_i \to zp\,,\ p_j \to (1-z)p$$

for all $n \geq m$

NLO complications

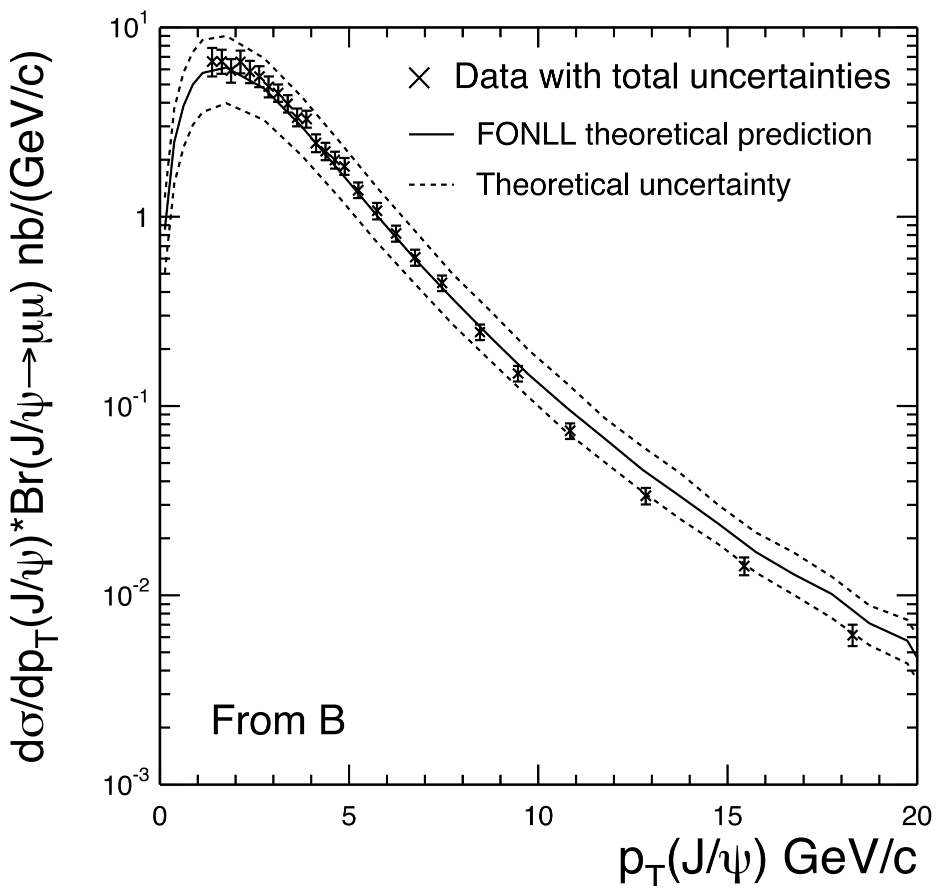
- loop integrals are involved and process-dependent
- more particles $\Rightarrow$ many scales $\Rightarrow$ lenghty analytic expressions
  - even though it is known how to compute loop integrals with $2 \to n$ particles no one-loop amplitudes with $n > 4$ have been computed (either analytically or numerically)
- no fully numeric methods yet for hadron collisions
  - counterterms are subtracted analytically

Is NLO enough to describe data ?

b cross section in $p\bar{p}$ collisions at 1.96 TeV

$$d\sigma(p\bar{p} \to H_b X, H_b \to J/\psi\ X)/dp_T(J/\psi)$$

CDF hep-ex/0412071



total x-sect is

$$19.4 \pm 0.3(stat)^{+2.1}_{-1.9}(syst)\ \mathrm{nb}$$

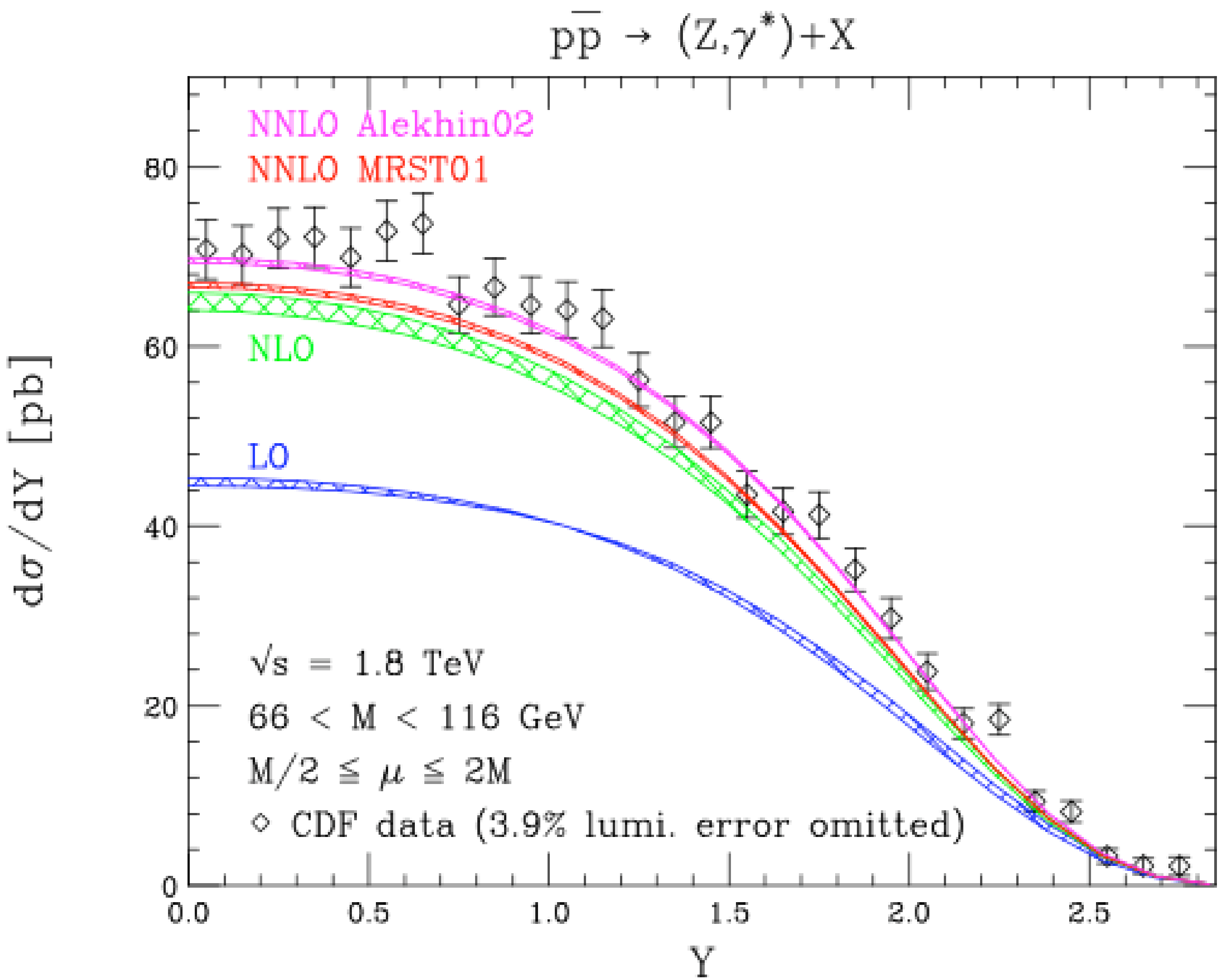
FONLL = NLO + NLL

Cacciari, Frixione, Mangano, Nason, Ridolfi 2003

good agreement with data (with use of updated FF's by Cacciari & Nason)

Is NLO enough to describe data ?

di-lepton rapidity distribution for (Z, γ^*) production vs. Tevatron Run I data



LO and NLO curves are for the MRST PDF set

no spin correlations

C. Anastasiou L. Dixon K. Melnikov F. Petriello 2003

Is NLO enough to describe data ?

Drell-Yan W cross section at LHC with leptonic decay of the W

Cuts A $\longrightarrow |\eta^{(e)}| < 2.5,\ p_T^{(e)} > 20$ GeV, $p_T^{(\nu)} > 20$ GeV

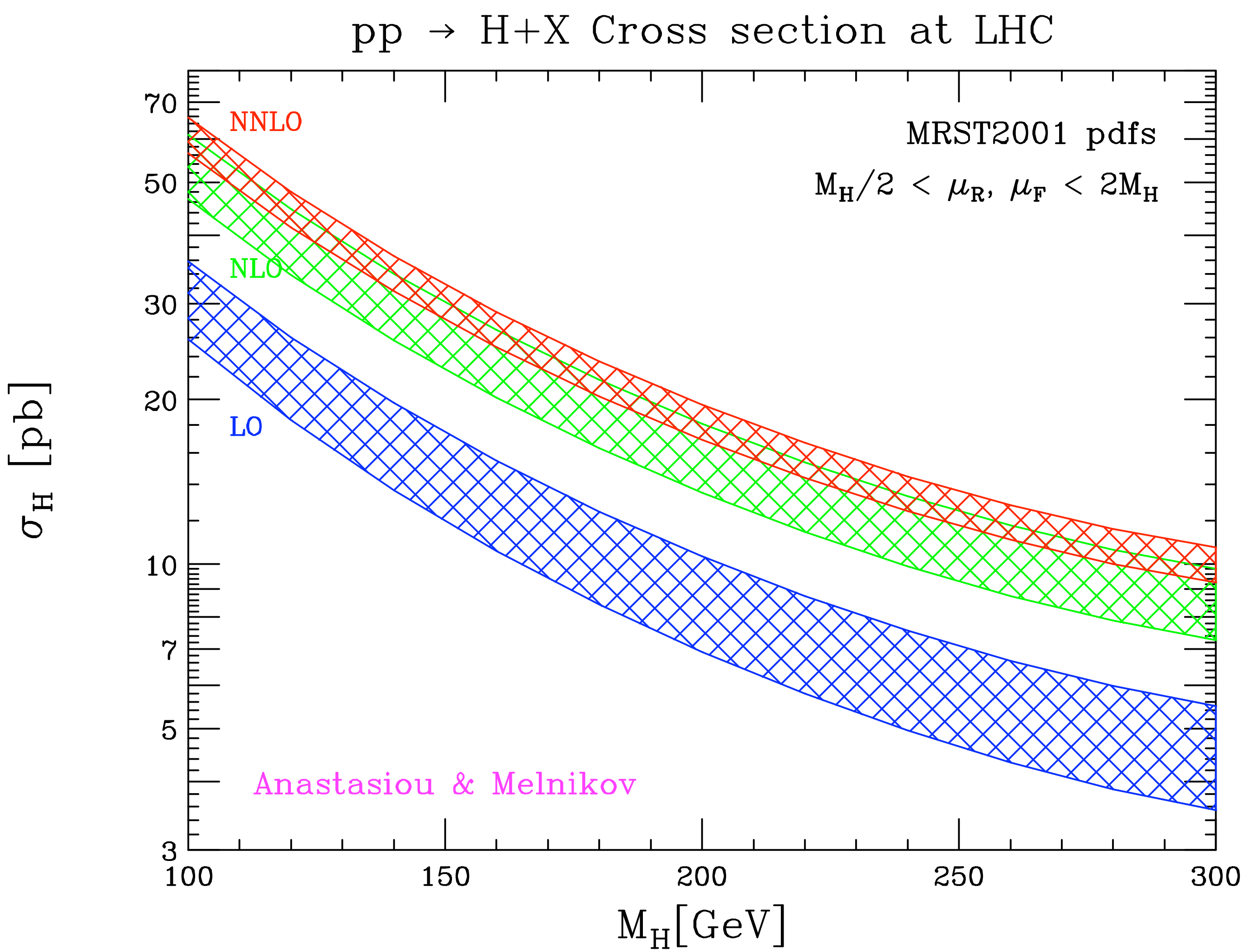
Cuts B $\longrightarrow |\eta^{(e)}| < 2.5,\ p_T^{(e)} > 40$ GeV, $p_T^{(\nu)} > 20$ GeV

	LO		LO+HW	NLO		MC@NLO
Cuts A	0.5249	$\xrightarrow{-7.7\%}$	0.4843	0.4771	$\xrightarrow{+1.5\%}$	0.4845
	↓5.4%			↓7.0%		↓6.3%
Cuts A, no spin	0.5535			0.5104		0.5151
Cuts B	0.0585	$\xrightarrow{+208\%}$	0.1218	0.1292	$\xrightarrow{+2.9\%}$	0.1329
	↓29%			↓16%		↓18%
Cuts B, no spin	0.0752			0.1504		0.1570

- $|\mathrm{MC@NLO} - \mathrm{NLO}| = \mathcal{O}(2\%)$ — S. Frixione M.L. Mangano 2004
- NNLO useless without spin correlations
- Precisely evaluated Drell-Yan W, Z cross sections could be used as ``standard candles'' to measure the parton luminosity at LHC

Is NLO enough to describe data ?

Total cross section for inclusive Higgs production at LHC



contour bands are

lower

$\mu_R = 2M_H \quad \mu_F = M_H/2$

upper

$\mu_R = M_H/2 \quad \mu_F = 2M_H$

scale uncertainty is about 10%

NNLO prediction stabilises the perturbative series

NNLO corrections may be relevant if

- the main source of uncertainty in extracting info from data is due to NLO theory: α_S measurements
- NLO corrections are large: Higgs production from gluon fusion in hadron collisions
- NLO uncertainty bands are too large to test theory vs. data: b production in hadron collisions
- NLO is effectively leading order: energy distributions in jet cones

in short, NNLO is relevant where NLO fails to do its job

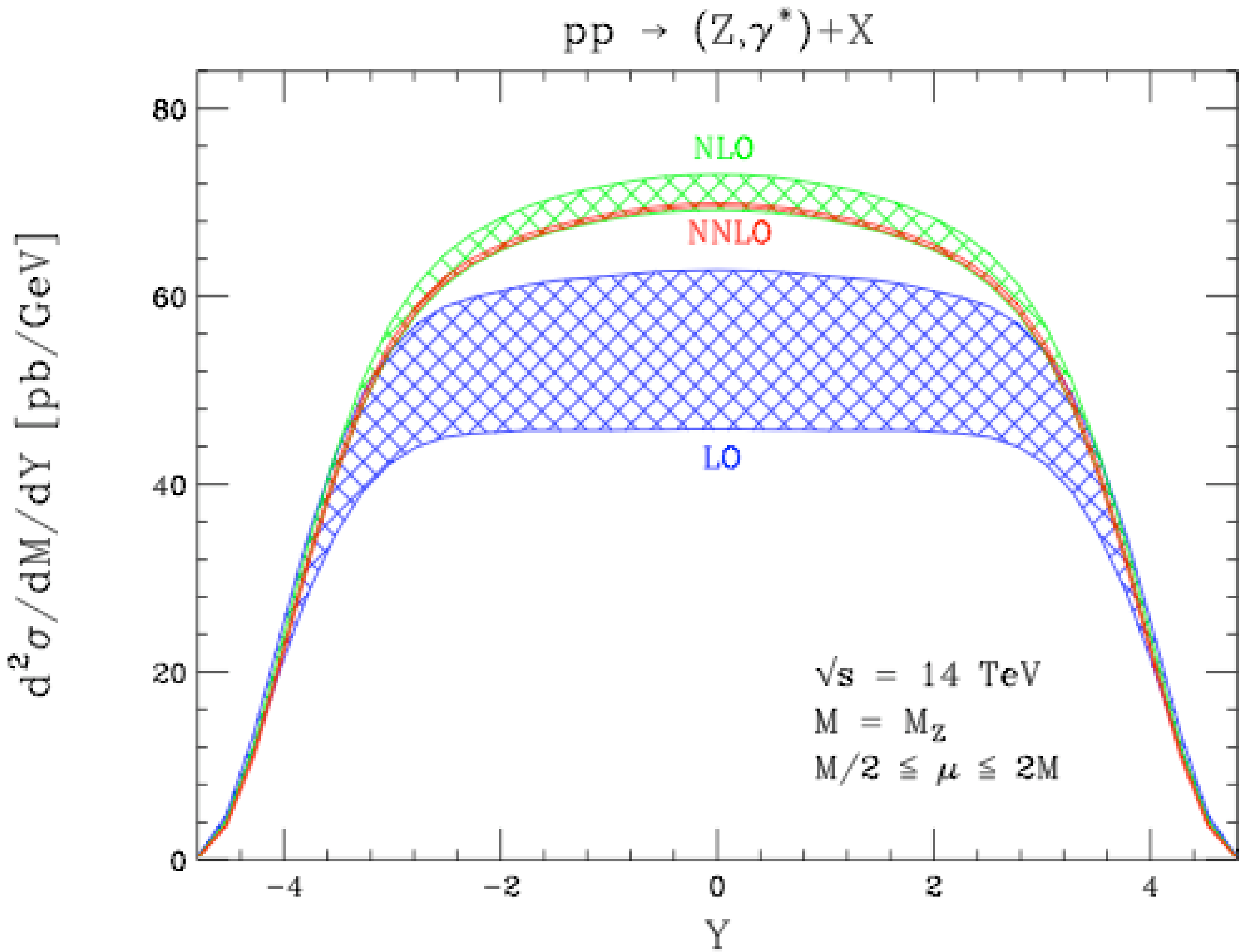
NNLO state of the art

- Drell-Yan W, Z production
  - total cross section — Hamberg, van Neerven, Matsuura 1990; Harlander, Kilgore 2002
  - rapidity distribution — Anastasiou Dixon Melnikov Petriello 2003
- Higgs production
  - total cross section — Harlander, Kilgore; Anastasiou, Melnikov 2002; Ravindran, Smith, van Neerven 2003
  - fully differential cross section — Anastasiou, Melnikov, Petriello 2004
- $e^+e^- \to 3$ jets
  - the $1/N_c^2$ term — the Gehrmanns, Glover 2004-5

NNLO cross sections

- **Analytic integration** — Hamberg, van Neerven, Matsuura 1990; Anastasiou Dixon Melnikov Petriello 2003
  - ↑ first method
  - ⇒ flexible enough to include a limited class of acceptance cuts by modelling cuts as ``propagators''
- **Sector decomposition** — Denner Roth 1996; Binoth Heinrich 2000; Anastasiou, Melnikov, Petriello 2004
  - ↑ flexible enough to include any acceptance cuts
  - ↑ cancellation of divergences is performed numerically
  - ⇒ can it handle many final-state partons ?
- **Subtraction**
  - ↑ process independent
  - ⇒ cancellation of divergences is semi-analytic can it be fully automatised ?

Drell-Yan Z production at LHC

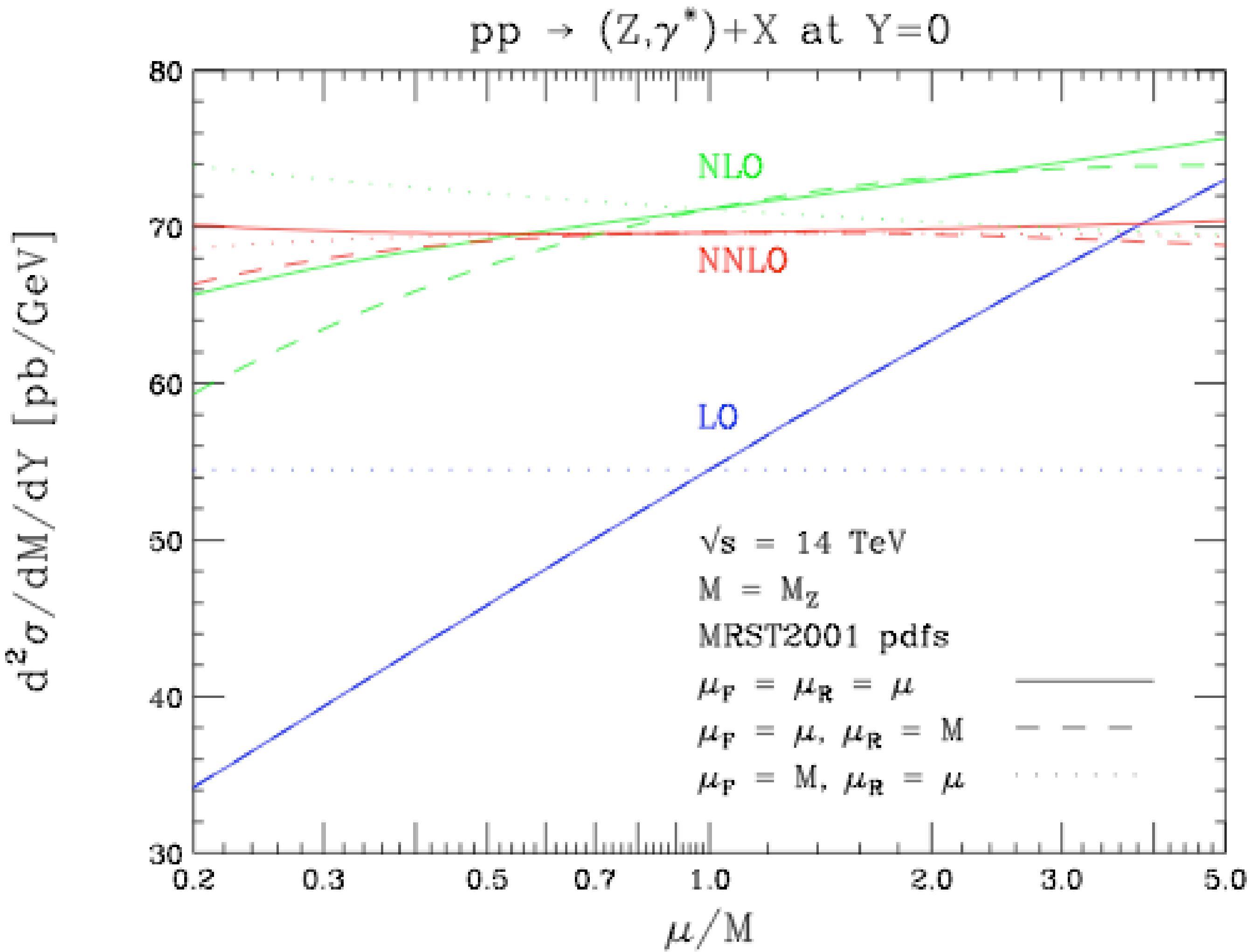


Rapidity distribution for an on-shell Z boson

- $30\%(15\%)$ NLO increase wrt to LO at central Y's (at large Y's) NNLO decreases NLO by $1-2\%$
- scale variation: $\approx 30\%$ at LO; $\approx 6\%$ at NLO; less than 1% at NNLO

C. Anastasiou L. Dixon K. Melnikov F. Petriello 2003

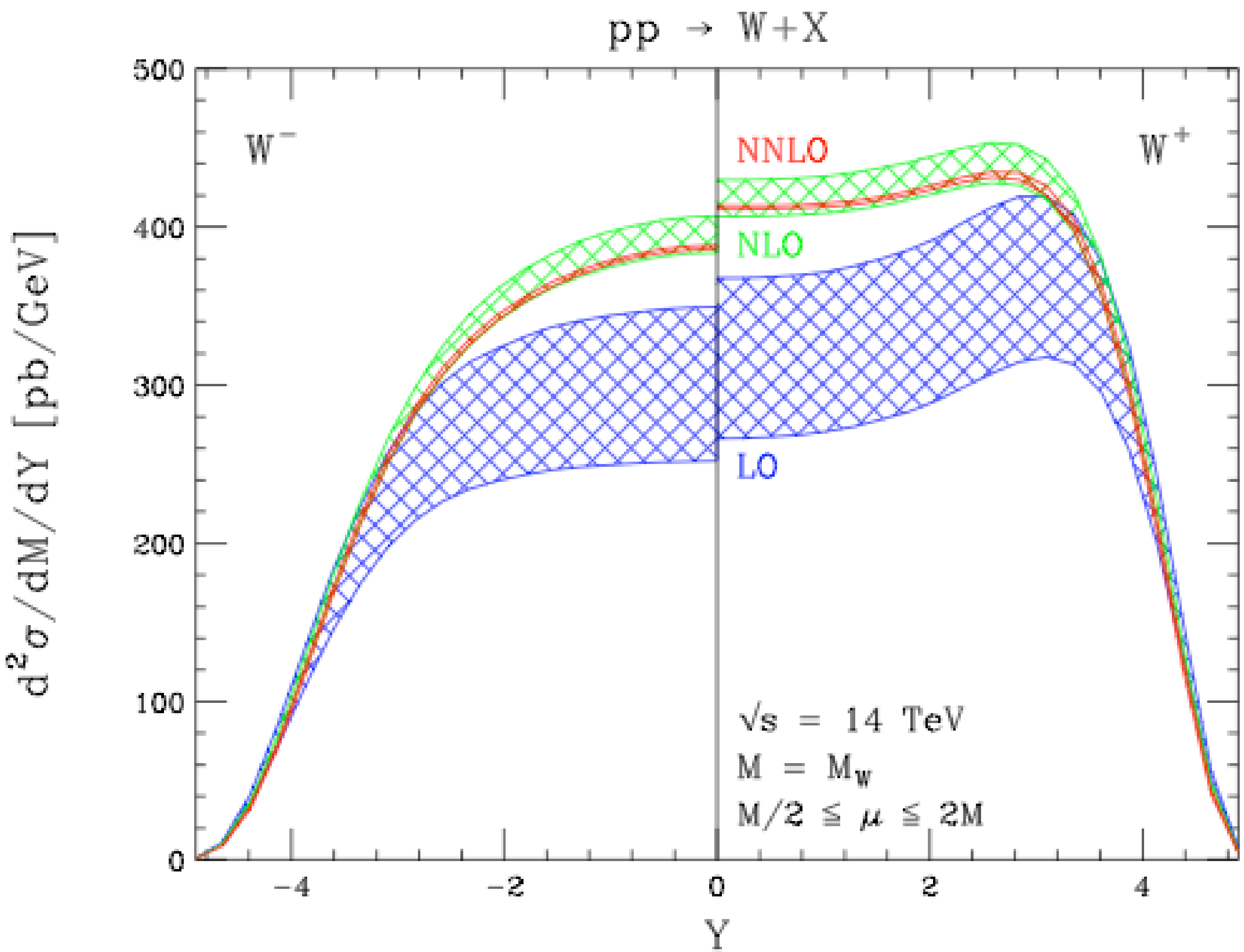
Scale variations in Drell-Yan Z production



- solid: vary μ_R and μ_F together
- dashed: vary μ_F only
- dotted: vary μ_R only

C. Anastasiou L. Dixon K. Melnikov F. Petriello 2003

Drell-Yan W production at LHC



Rapidity distribution for an on-shell W^- boson (left) W^+ boson (right)

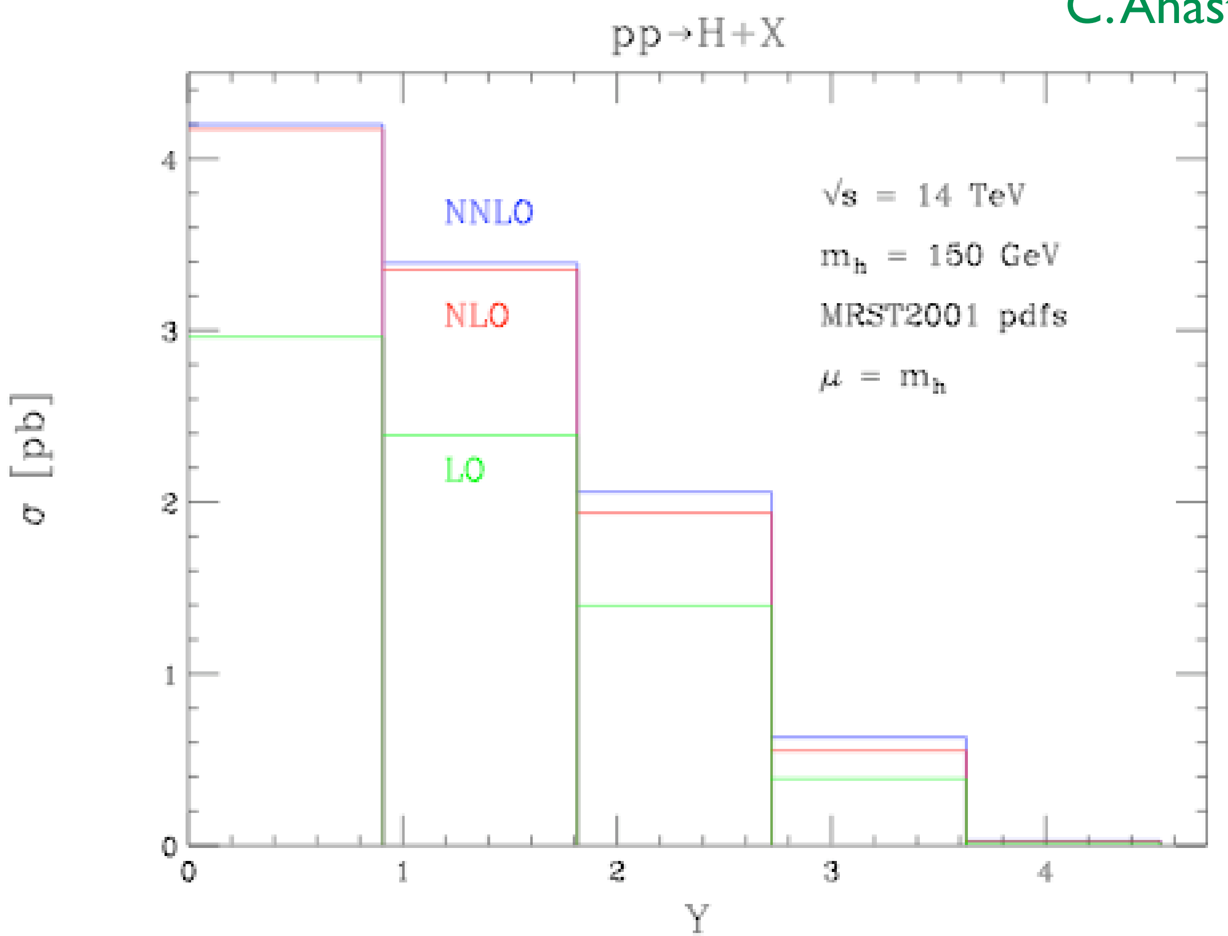
- distributions are symmetric in Y
- NNLO scale variations are $1\%(3\%)$ at central (large) Y

C. Anastasiou L. Dixon K. Melnikov F. Petriello 2003

Higgs production at LHC

a fully differential cross section:
bin-integrated rapidity distribution, with a jet veto

C. Anastasiou K. Melnikov F. Petriello 2004



jet veto: require

$R = 0.4$

$|\mathbf{p}_T^j| < p_T^{veto} = 40 \text{ GeV}$

for 2 partons

$R_{12}^2 = (\eta_1 - \eta_2)^2 + (\phi_1 - \phi_2)^2$

if $R_{12} > R$

$|\mathbf{p}_T^1|, |\mathbf{p}_T^2| < p_T^{veto}$

if $R_{12} < R$

$|\mathbf{p}_T^1 + \mathbf{p}_T^2| < p_T^{veto}$

- $M_H = 150 \text{ GeV}$ (jet veto relevant in the $H \to W^+W^-$ decay channel)
- K factor is much smaller for the vetoed x-sect than for the inclusive one: average $|\mathbf{p}_T^j|$ increases from NLO to NNLO: less x-sect passes the veto

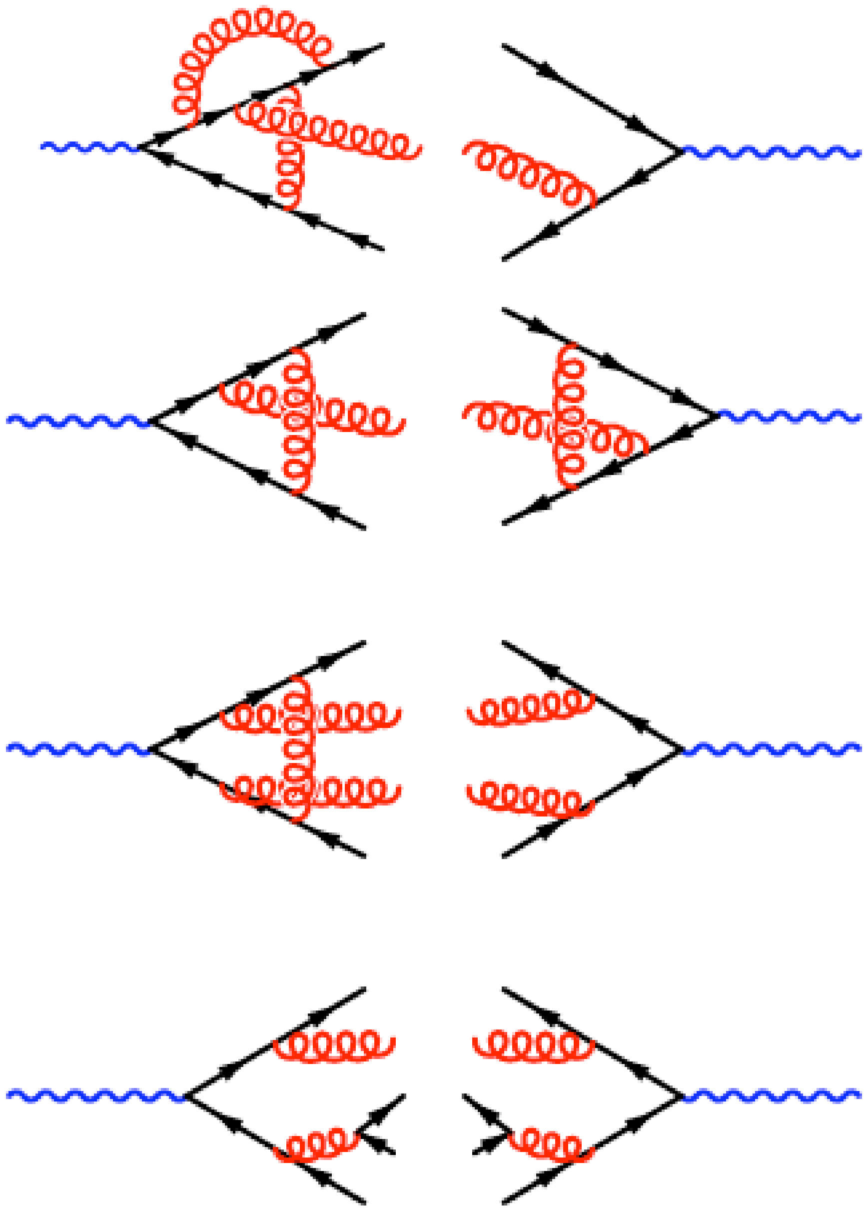
NNLO assembly kit

$e^+e^- \rightarrow 3$ jets

double virtual

real-virtual

double real



Two-loop matrix elements

- two-jet production $qq' \to qq', \ q\bar{q} \to q\bar{q}, \ q\bar{q} \to gg, \ gg \to gg$

  C. Anastasiou N. Glover C. Oleari M. Tejeda-Yeomans 2000-01

  Z. Bern A. De Freitas L. Dixon 2002

- photon-pair production $q\bar{q} \to \gamma\gamma, \ gg \to \gamma\gamma$

  C. Anastasiou N. Glover M. Tejeda-Yeomans 2002
  Z. Bern A. De Freitas L. Dixon 2002

- $e^+e^- \to 3$ jets $\gamma^* \to q\bar{q}g$

  L. Garland T. Gehrmann N. Glover A. Koukoutsakis E. Remiddi 2002

- $V + 1$ jet production $q\bar{q} \to Vg$

  T. Gehrmann E. Remiddi 2002

- Drell-Yan V production $q\bar{q} \to V$

  R. Hamberg W. van Neerven T. Matsuura 1991

- Higgs production $gg \to H$ (in the $m_t \to \infty$ limit)

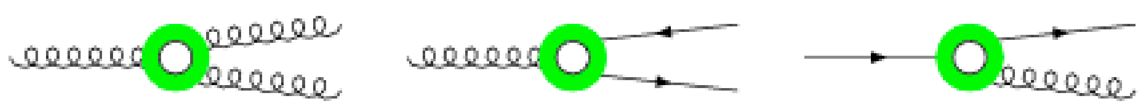
  R. Harlander W. Kilgore; C. Anastasiou K. Melnikov 2002

NNLO cross sections

- universal IR structure ⟹ process-independent procedure
- universal collinear and soft currents
  - 3-parton tree splitting functions

J. Campbell N. Glover 1997; S. Catani M. Grazzini 1998; A. Frizzo F. Maltoni VDD 1999; D. Kosower 2002

  - 2-parton one-loop splitting functions



Z. Bern L. Dixon D. Dunbar D. Kosower 1994;
Z. Bern W. Kilgore C. Schmidt VDD 1998-99;
D. Kosower P. Uwer 1999; D. Kosower 2003

- universal subtraction counterterms
  - several ideas and works in progress

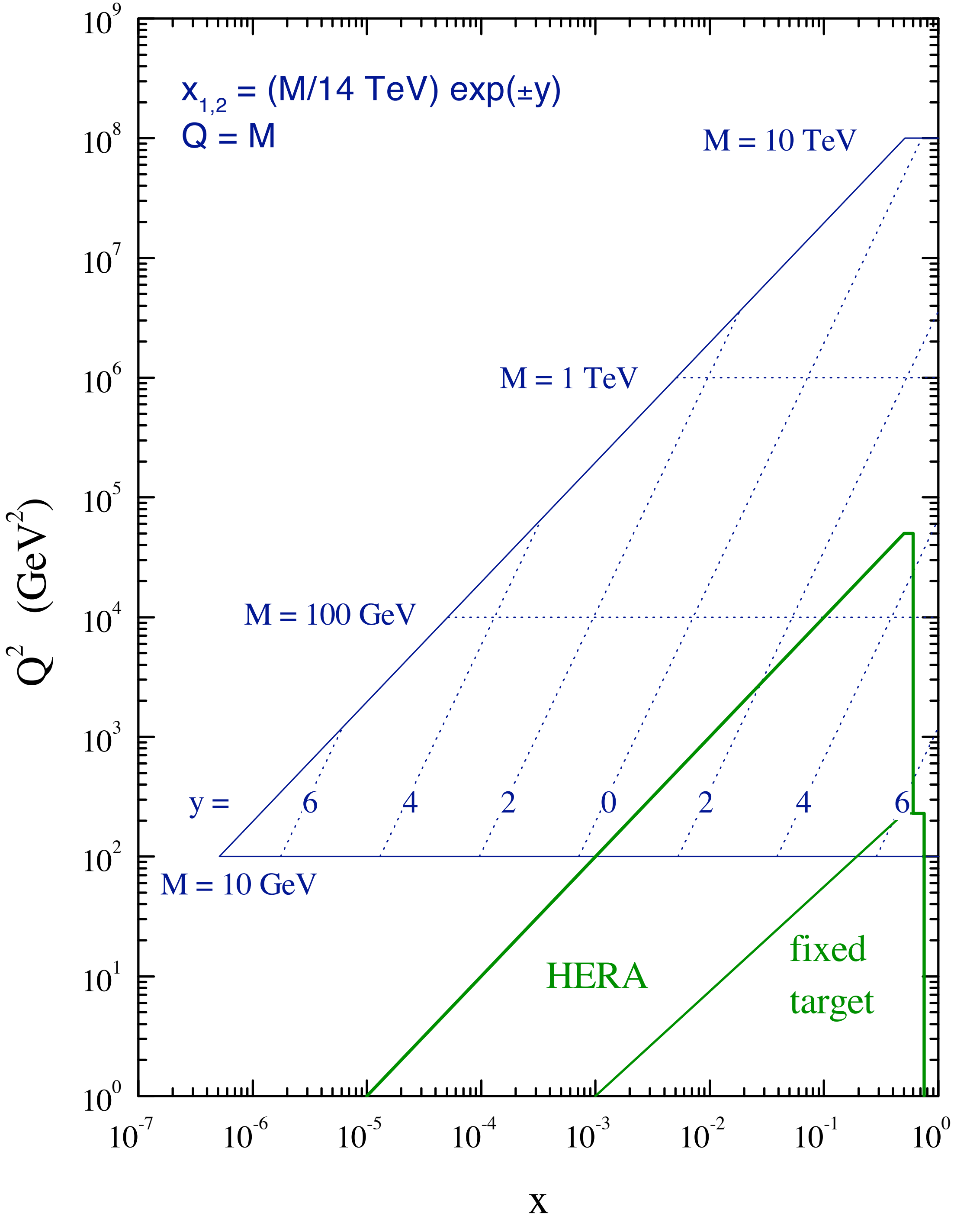
D. Kosower; S. Weinzierl; the Gehrmanns & G. Heinrich 2003
S. Frixione M. Grazzini 2004; G. Somogyi Z. Trocsanyi VDD 2005

  - but completely figured out only for $e^+e^- \to 3$ jets

the Gehrmanns & N. Glover 2005

LHC parton kinematics

J. Stirling



Evolution

- factorisation scale μ_F is arbitrary
  - cross section cannot depend on μ_F

$$\mu_F \frac{d\sigma}{d\mu_F} = 0$$

implies DGLAP equations

V. Gribov L. Lipatov; Y. Dokshitzer
G. Altarelli G. Parisi

$$\mu_F \frac{df_a(x,\mu_F^2)}{d\mu_F} = P_{ab}(x,\alpha_S(\mu_F^2)) \otimes f_b(x,\mu_F^2) + \mathcal{O}(\frac{1}{Q^2})$$

$$\mu_F \frac{d\hat{\sigma}_{ab}(Q^2/\mu_F^2,\alpha_S(\mu_F^2))}{d\mu_F} = -P_{ac}(x,\alpha_S(\mu_F^2)) \otimes \hat{\sigma}_{cb}(Q^2/\mu_F^2,\alpha_S(\mu_F^2)) + \mathcal{O}(\frac{1}{Q^2})$$

  - $P_{ab}(x,\alpha_S(\mu_F^2))$ is calculable in pQCD

Parton distribution functions (PDF)

- factorisation for the structure functions (e.g. $F_2^{ep}, \; F_L^{ep}$)

$$\mathcal{F}_i(x,\mu_F^2) = C_{ij}\otimes q_j + C_{ig}\otimes g$$

with the convolution $$[a\otimes b](x) \;\equiv\; \int_x^1 \frac{dy}{y}\, a(y)\, b\!\left(\frac{x}{y}\right)$$

$C_{ij}, \; C_{ig}$ coefficient functions

$q_i(x,\mu_F^2) \quad g(x,\mu_F^2)$ PDF's

- DGLAP evolution equations

$$\frac{d}{d\ln\mu_F^2}\begin{pmatrix} q_i \\ g \end{pmatrix} = \begin{pmatrix} P_{\mathrm{q_i q_j}} & P_{\mathrm{q_j g}} \\ P_{\mathrm{g q_j}} & P_{\mathrm{gg}} \end{pmatrix} \otimes \begin{pmatrix} q_j \\ g \end{pmatrix}$$

- perturbative series $P_{ij} \approx \alpha_s P_{ij}^{(0)} + \alpha_s^2 P_{ij}^{(1)} + \alpha_s^3 P_{ij}^{(2)}$

- anomalous dimension $\gamma_{ij}(N) = -\int_0^1 dx\; x^{N-1}\; P_{ij}(x)$

PDF's

- general structure of the quark-quark splitting functions

$$P_{\mathrm{q}_i\mathrm{q}_k} = P_{\bar{\mathrm{q}}_i\bar{\mathrm{q}}_k} = \delta_{ik} P^{\mathrm{v}}_{\mathrm{qq}} + P^{\mathrm{s}}_{\mathrm{qq}}$$
$$P_{\mathrm{q}_i\bar{\mathrm{q}}_k} = P_{\bar{\mathrm{q}}_i\mathrm{q}_k} = \delta_{ik} P^{\mathrm{v}}_{\mathrm{q}\bar{\mathrm{q}}} + P^{\mathrm{s}}_{\mathrm{q}\bar{\mathrm{q}}}$$

- flavour non-singlet
  - flavour asymmetry

$$q^{\pm}_{\mathrm{ns},ik} = q_i \pm \bar{q}_i - (q_k \pm \bar{q}_k) \quad \Longleftarrow \quad P^{\pm}_{\mathrm{ns}} = P^{\mathrm{v}}_{\mathrm{qq}} \pm P^{\mathrm{v}}_{\mathrm{q}\bar{\mathrm{q}}}$$

  - sum of valence distributions of all flavours

$$q^{\mathrm{v}}_{\mathrm{ns}} = \sum_{r=1}^{n_f} (q_r - \bar{q}_r) \quad \Longleftarrow \quad P^{\mathrm{v}}_{\mathrm{ns}} = P^{\mathrm{v}}_{\mathrm{qq}} - P^{\mathrm{v}}_{\mathrm{q}\bar{\mathrm{q}}} + n_f (P^{\mathrm{s}}_{\mathrm{qq}} - P^{\mathrm{s}}_{\mathrm{q}\bar{\mathrm{q}}})$$

- flavour singlet

$$q_{\mathrm{s}} = \sum_{i=1}^{n_f} (q_i + \bar{q}_i) \quad \Longleftarrow \quad \frac{d}{d \ln \mu_F^2} \begin{pmatrix} q_{\mathrm{s}} \\ g \end{pmatrix} = \begin{pmatrix} P_{\mathrm{qq}} & P_{\mathrm{qg}} \\ P_{\mathrm{gq}} & P_{\mathrm{gg}} \end{pmatrix} \otimes \begin{pmatrix} q_{\mathrm{s}} \\ g \end{pmatrix}$$

with

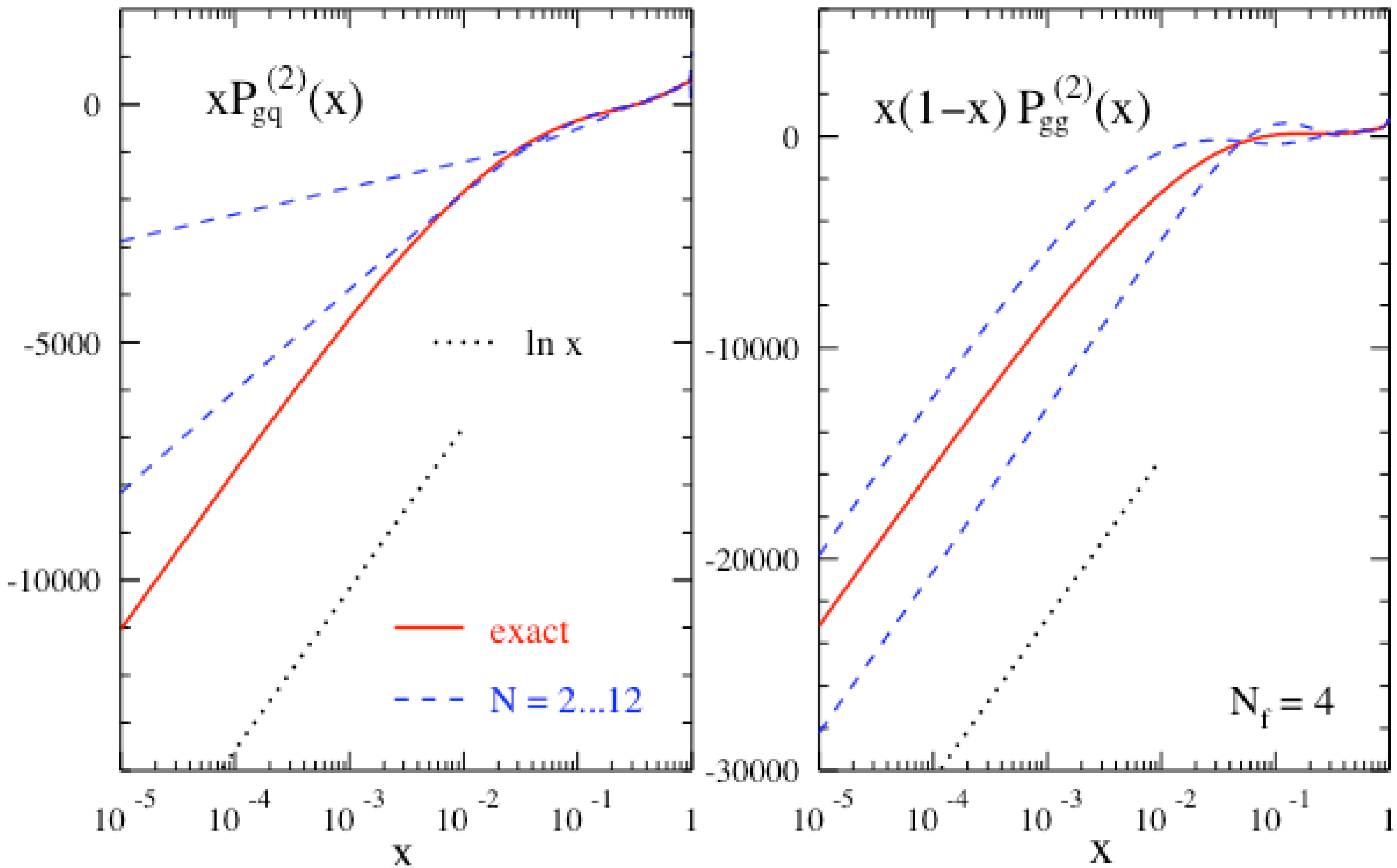
$$P_{\mathrm{qq}} = P^{+}_{\mathrm{ns}} + n_f (P^{\mathrm{s}}_{\mathrm{qq}} + P^{\mathrm{s}}_{\bar{\mathrm{q}}\mathrm{q}})$$
$$P_{\mathrm{qg}} = n_f P_{\mathrm{q}_i\mathrm{g}} \,, \qquad P_{\mathrm{gq}} = P_{\mathrm{gq}_i}$$

PDF history

- leading order (or one-loop) anomalous dim/splitting functions — Gross Wilczek 1973; Altarelli Parisi 1977
- NLO (or two-loop)
  F_2, F_L — Bardeen Buras Duke Muta 1978
  anomalous dim/splitting functions — Curci Furmanski Petronzio 1980
- NNLO (or three-loop)
  F_2, F_L — Zijlstra van Neerven 1992; Moch Vermaseren 1999
  anomalous dim/splitting functions — Moch Vermaseren Vogt 2004
  - the calculation of the three-loop anomalous dimension is the toughest calculation ever performed in perturbative QCD!
    - one-loop $\gamma_{ij}^{(0)}/P_{ij}^{(0)}$ ⇒ 18 Feynman diagrams
    - two-loop $\gamma_{ij}^{(1)}/P_{ij}^{(1)}$ ⇒ 350 Feynman diagrams
    - three-loop $\gamma_{ij}^{(2)}/P_{ij}^{(2)}$ ⇒ 9607 Feynman diagrams

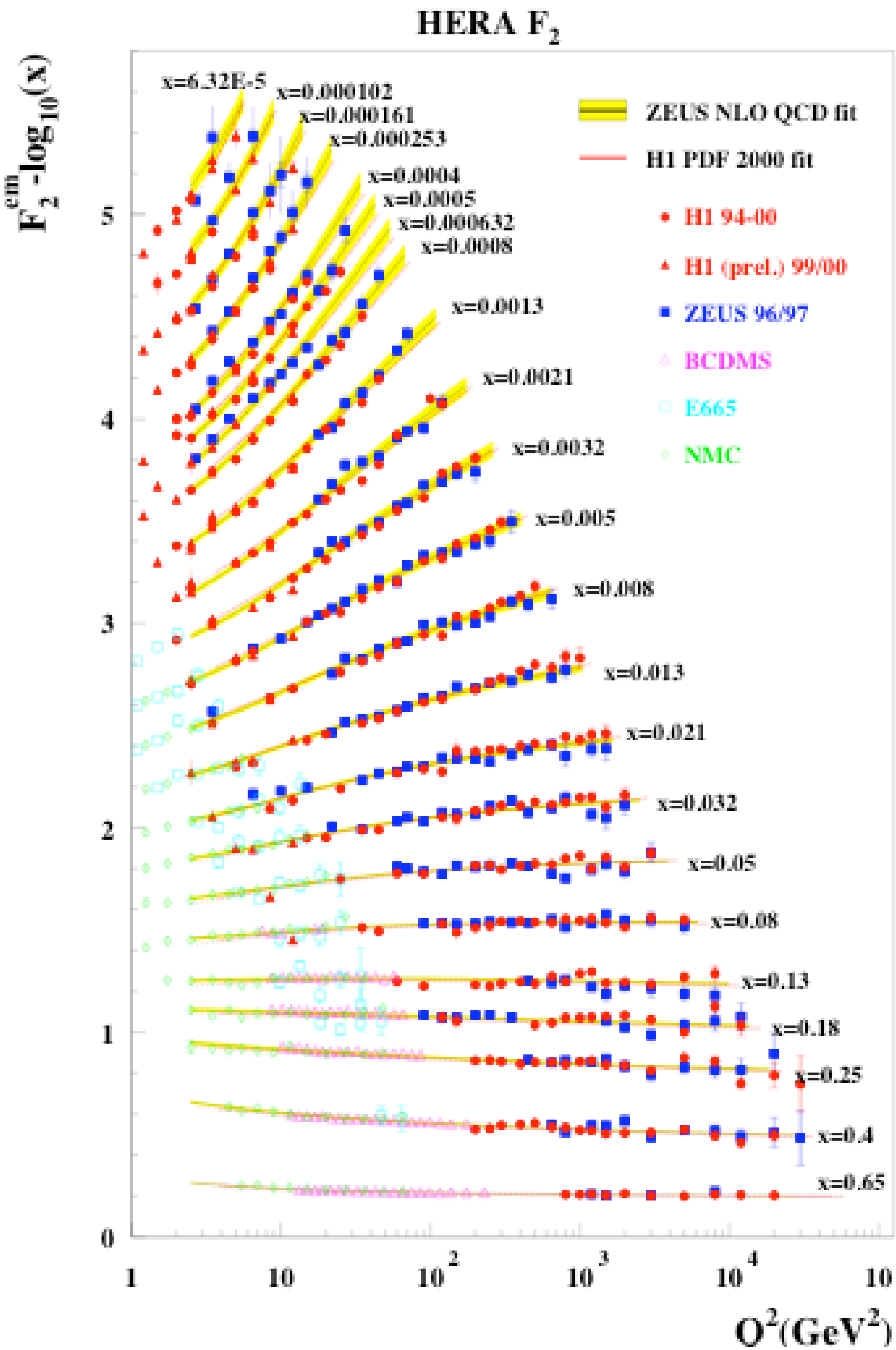
    20 man-year-equivalents, 10^6 lines of dedicated algebra code

Numerical examples



- exact NNLO results, estimates from fixed moments and leading small-x term

HERA F_2



Bjorken-scaling violations

H1, ZEUS: ongoing fits for PDF's; so far NNLO not included

PDF global fits

J. Stirling, KITP collider conf 2004

global fits

MRST: Martin Roberts Stirling Thorne

CTEQ: Pumplin et al.

Alekhin (DIS data only)

method

Perform fit by minimising χ^2 to all data, including both statistical and systematic errors

Start evolution at some Q_0^2, where PDF's are parametrised with functional form, e.g.

$$xf(x,Q_0^2) = (1-x)^\eta(1+\epsilon x^{0.5}+\gamma x)x^\delta$$

Cut data at $Q^2 > Q^2_{\min}$ and at $W^2 > W^2_{\min}$ to avoid higher twist contamination

Allow $\bar{u} \neq \bar{d}$ as implied by E866 Drell-Yan asymmetry data

accuracy

NLO evolution
and fixed moments of NNLO

H1, ZEUS $F_2^{e^+p}(x,Q^2)$, $F_2^{e^-p}(x,Q^2)$

BCDMS $F_2^{\mu p}(x,Q^2)$, $F_2^{\mu d}(x,Q^2)$

NMC $F_2^{\mu p}(x,Q^2)$, $F_2^{\mu d}(x,Q^2)$, $(F_2^{\mu n}(x,Q^2)/F_2^{\mu p}(x,Q^2))$

SLAC $F_2^{\mu p}(x,Q^2)$, $F_2^{\mu d}(x,Q^2)$

E665 $F_2^{\mu p}(x,Q^2)$, $F_2^{\mu d}(x,Q^2)$

CCFR $F_2^{\nu(\bar{\nu})p}(x,Q^2)$, $F_3^{\nu(\bar{\nu})p}(x,Q^2)$

$\rightarrow q,\ \bar{q}$ at all x and g at medium, small x

H1, ZEUS $F_{2,c}^{e^+p}(x,Q^2) \rightarrow c$

E605, E772, E866 Drell-Yan $pN \rightarrow \mu\bar{\mu} + X \rightarrow \bar{q}\ (g)$

E866 Drell-Yan p,n asymmetry $\rightarrow \bar{u}, \bar{d}$

CDF W rapidity asymmetry $\rightarrow u/d$ ratio at high x

CDF, D0 Inclusive jet data $\rightarrow g$ at high x

CCFR, NuTeV Dimuon data constrains strange sea $s,\ \bar{s}$

- no prompt photon data included in the fits

Conclusions

- QCD is an extensively developed and tested gauge theory
- a lot of progress in the last 4-5 years in
  - MonteCarlo generators
  - NLO cross sections with one more jet
  - NNLO computations
- better and better approximations of signal and background for Higgs and New Physics